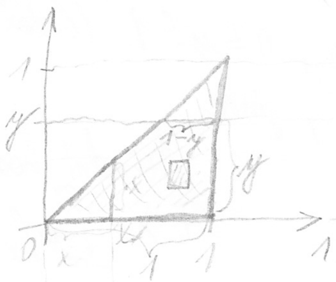
Př.: (X,Y) má rovnoměrné rozdělení na Δ: $(0,0)$, $(1,0)$, $(1,1)$.

Popište a znázorněte

distr. fce X, Y, (X,Y)

marginální sdružená

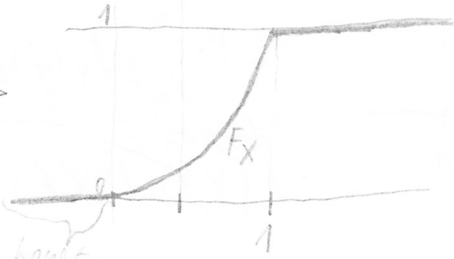


Marginální: (bod na odvěsně Δ)

$$F_X(x) = P[X \le x] = \frac{\frac{x^2}{2}}{\frac{1}{2}} = x^2$$

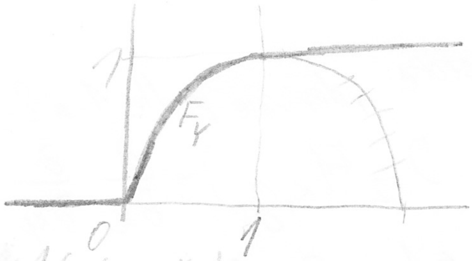
... malý Δ / obsah celého Δ

⇒ parabola ⇒



$$F_Y(y) = P[Y \le y] = y\,\frac{\frac{1+(1-y)}{2}}{\frac{1}{2}} = y(2-y)$$

... lichoběžník



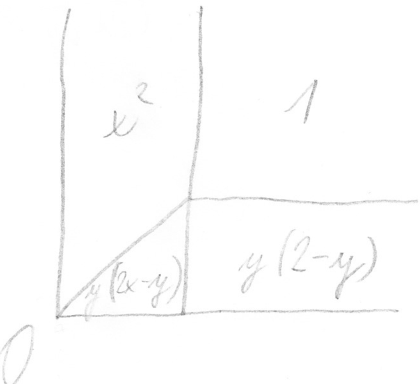
$$F_{X,Y}(x,y) = P[X \le x, Y \le y] =$$

(tj. $X \le x \wedge Y \le y$) AND

vymezuje Δ (celý velký)

$$= \begin{cases} y\,\frac{x+(x-y)}{2}\cdot 2 = y(2x-y), & 0 \le y \le x \le 1 \\ x^2, & 0 \le x < y,\ x \le 1 \\ 0, & x < 0 \vee y < 0 \\ y(2-y), & 0 \le y \le 1 < x \\ 1, & x > 1,\ y > 1 \end{cases}$$

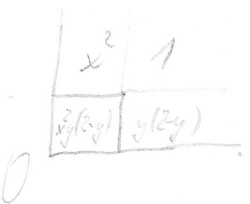
⇒



Pokud by X, Y byly nezávislé: sdružená je součin marginálních

$$F_{XY}(x,y) = F_X(x) \cdot F_Y(y) = P[X \le x] \cdot P[Y \le y] =$$

$$= x^2 y(2-y) \quad \text{pro } x, y \in \langle 0, 1 \rangle$$



pro $x = y = \frac{1}{2}$: $F_{XY}\left(\frac{1}{2},\frac{1}{2}\right) = x^2 y(2-y) = \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} \cdot \left(2 - \frac{1}{2}\right) = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{16} = F_X\left(\frac{1}{2}\right) \cdot F_Y\left(\frac{1}{2}\right) = \frac{1}{4} \cdot \frac{1}{2}\left(2 - \frac{1}{2}\right)$ ✓

podle původ. vzorce $F_{XY}\left(\frac{1}{2},\frac{1}{2}\right) = \frac{1}{2}\left(2 \cdot \frac{1}{2} - \frac{1}{2}\right) = \frac{1}{4} \neq \frac{3}{16}$!! ⇒

⇒ původní NV byly ZÁVISLÉ

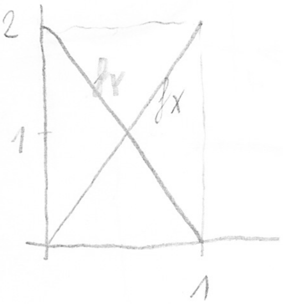
Hustota

$$f_X(x) = \begin{cases} 2x & , \ 0 \le x \le 1 \\ 0 & , \ \text{jinde} \end{cases}$$

sice nemá v $x=1$ derivaci, ale nevadí – 1 bod si můžu dodefinovat libovolně

$$f_Y(y) = \begin{cases} 2-2y & , \ 0 \le y \le 1 \\ 0 & , \ \text{jinde} \end{cases}$$

(totéž v 0)



Střední hodnota

$(-\infty;0)$ a $(1;\infty)$ zahodím, protože

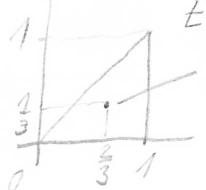
$$EX = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_0^1 2x^2\,dx = \frac{2}{3}\left[x^3\right]_{x=0}^{1} = \frac{2}{3}$$

$$EY = \int_{-\infty}^{\infty} y f_Y(y)\,dy = \int_0^1 2y(1-y)\,dy = \left[y^2 - \frac{2}{3}y^3\right]_{y=0}^{1} = 1 - \frac{2}{3} = \frac{1}{3}$$

Sdružená E:

$$E(X,Y) = (EX, EY) = \left(\frac{2}{3}, \frac{1}{3}\right) \ldots$$

E zjistíme i z obrázku: těžiště Δ



Sdružená hustota:

derivace podle 1. a 2. proměnné

$$f_{X,Y}(x,y) = D_{1,2}F(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial y}F(x,y)$$

$0 \le y \le x \le 1$: $\quad \frac{\partial}{\partial x}\frac{\partial}{\partial y}\, y(2x-y) = \frac{\partial}{\partial y} 2y = 2$

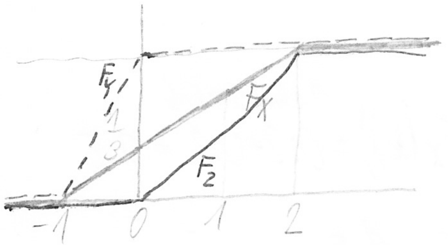
jinak: $\quad 0$

$\Big\}\ f_{X,Y}(x,y)$

$\left(\frac{\partial}{\partial x}\frac{\partial}{\partial y}\right.$ všechny ostatní 4 případy „vynuluje")

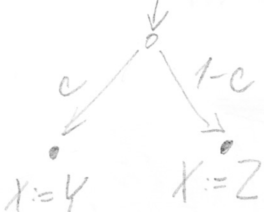
Př.: Směs pstí

X rovnoměrné rozdělení od −1 do 2



$X = Mix_c(Y, Z)$

$Y \le 0 \quad Z \ge 0$



$P[Y \ge 0.001] = 0$

$F_Y(0.001) = Y \le \quad = 1$

$F_X(t) = c \; F_Y(t) + (1-c) \; F_Z(t)$

$F_X(0) = c \; F_Y(0) + (1-c) \; F_Z(0)$

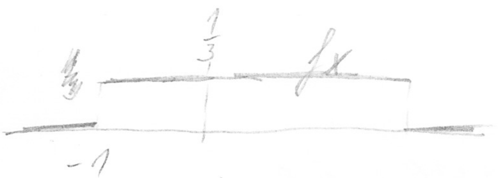
$\frac{1}{3} = c \cdot 1 + (1-c) \cdot 0 \Rightarrow c = \frac{1}{3}$

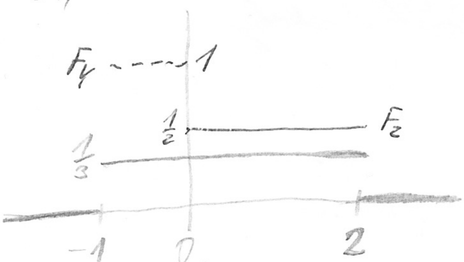
$\frac{t+1}{3} = F_X(t) = \frac{1}{3} \underbrace{F_Y(t)}_{} + \frac{2}{3} F_Z(t)$

$0 \le t \le 2$: $= 1 \Rightarrow F_Z(t) = \left(\frac{t+1}{3} - \frac{1}{3}\right)\frac{3}{2} = \frac{t}{2}$

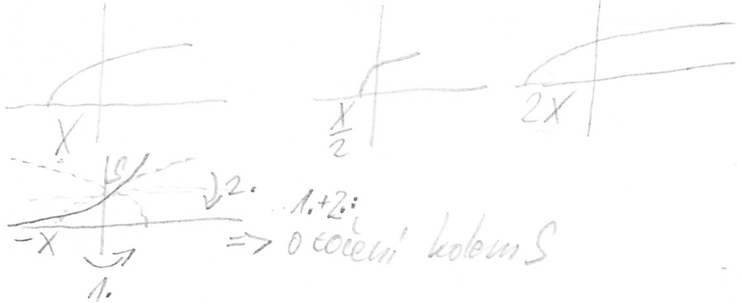
$$F_X(t) = \begin{cases} \frac{t+1}{3} & ; \ -1 \le t \le 2 \\ 1 & ; \ t > 2 \\ 0 & ; \ t < -1 \end{cases}$$

$-1 \le t \le 0$: $\frac{t+1}{3} = \frac{1}{3} F_Y(t) + 0 \Rightarrow F_Y(t) = t + 1$





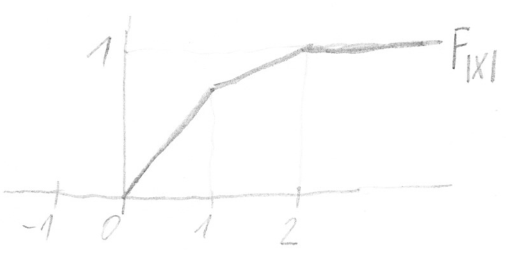
Operace s distr. f



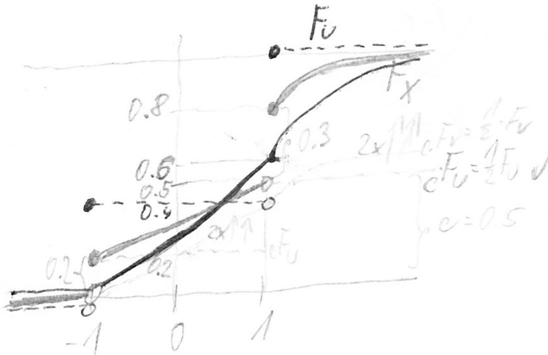
1.+2.: ⇒ otočení kolem S

Distr. fce abs. hod.

$F_{|X|} = Mix_c(|Y|, |Z|) = Mix_c(-Y, Z)$



Př.: Směs diskrétní a spojité NV



$X = \mathrm{Mix}_c(U, V)$

U diskr., obor hodnot $\{-1; 1\}$

V spoj.

$$c = \sum_t \left(\lim_{x \to t_+} F_X(x) - \lim_{x \to t_-} F_X(x) \right) = \underset{\text{1. skok}}{0.2} + \underset{\text{2. skok}}{0.3} = 0.5$$

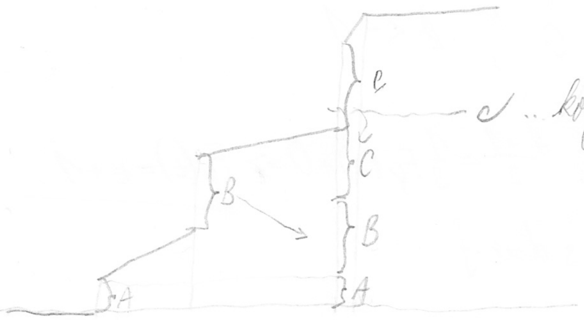
$$F_X = cF_U + (1-c)F_V = \frac{1}{2}F_U + \frac{1}{2}F_V$$

$$\Rightarrow \frac{1}{2}F_V = F_X - \frac{1}{2}F_U$$

$$F_V = 2F_X - F_U$$

lze kombinovat, co jsme dostali zvlášť pro diskr. a pro spoj.,
ale ne u věrohodnosti

Př.:



c ... koef. u diskr. složky - ~~součet~~ ~~skoků~~